<table>
<tr><th colspan="3">TOPIC PLAN</th></tr>
<tr><td>Partner organization</td><td colspan="2">UNS</td></tr>
<tr><td>Topic</td><td colspan="2">Function of Several Variables</td></tr>
<tr><td>Lesson title</td><td colspan="2">Application of Partial Derivatives</td></tr>
<tr><td>Learning objectives</td><td>✓ Students will be recall their knowledge zbout partial derivatives from the previous lesson
✓ Students will be introduced to various applications of partial derivatives and practise parial derivative calculation
✓ Students are encouraged to check their results using various online tools and also find the limits up to which internet resources can help them.</td><td rowspan="3">Strategies/Activities
☐Graphic Organizer
☐ Think/Pair/Share
☐ Modeling
☐Collaborative learning
☐Discussion questions
☐Project based learning
☐Problem based learning

Assessment for learning
☐Observations
☐Conversations
☐Work sample
☐Conference
☐Check list
☐Diagnostics

Assessment as</td></tr>
<tr><td>Aim of the lecture / Description of the practical problem</td><td>1. Find the tangent plane equation
2. Recognizing the use of partial derivatives in real time problems</td></tr>
<tr><td>Previous knowledge assumed:</td><td>- Basic calculus
- Partial derivatives
- differentiating techniques</td></tr>
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Introduction / Theoretical basics	content	learning

Introduction / Theoretical basics

An Economics Application: The Cobb–Douglas Production Function

One model of production that is frequently considered in business and economics is the Cobb–Douglas production function:

$$p(x,y) = Ax^a y^{1-a}, \text{ for } A > 0 \text{ and } 0 < a < 1,$$

where p is the number of units produced with x units of labor and y units of capital. (Capital is the cost of machinery, buildings, tools, and other supplies.) The partial derivatives

$$\frac{\partial p}{\partial x} \text{ and } \frac{\partial p}{\partial y}$$

are called, respectively, the *marginal productivity of labor* and the *marginal productivity of capital*.

EXAMPLE 2 A cellular phone company has the following production function for a smart phone:

$$p(x,y) = 50x^{2/3}y^{1/3}$$

where p is the number of units produced with x units of labor and y units of capital.

a) Find the number of units produced with 125 units of labor and 64 units of capital.
b) Find the marginal productivities.
c) Evaluate the marginal productivities at and $x = 125$ and $y = 64$.

Solution

a) $p(125,64) = 50(125)^{\frac{2}{3}}(64)^{1/3} = 5000$ units

b) Marginal productivity of labor is $\frac{\partial p}{\partial x} = p_x = 50\frac{2}{3}x^{-1/3}y^{1/3} = \frac{100y^{1/3}}{3x^{1/3}}$,

Marginal productivity of capital is

$\frac{\partial p}{\partial y} = p_y = 50\frac{1}{3}x^{2/3}y^{-2/3} = \frac{50x^{2/3}}{3y^{2/3}}$

c) For 125 units of labor and 64 units of capital, we have

$$p_x(125,64) = 26\frac{2}{3} \text{ and } p_y(125,64) = 26\frac{1}{24}$$

A Cobb–Douglas production function is consistent with the law of diminishing returns. That is, if one input (either labor or capital) is held fixed while the other increases infinitely, then production will eventually increase at a decreasing rate. With such functions, it also turns out that if a certain maximum production is possible, then the expense of more labor, for example, may be required for that maximum output to be attainable.

TANGENT PLANES AND LINEAR APPROXIMATIONS

Earlier we saw how the two partial derivatives f_x and f_y can be thought of as the slopes of traces. We want to extend this idea out a little in this section.

learning
☐Self-assessment
☐Peer-assessment
☐Presentation
☐Graphic Organizer
☐Homework

Assessment of learning
☐Test
☐Quiz
☐Presentation
☐Project
☐Published work

The graph of a function $z = f(x, y)$ is a surface in three dimensional space and so we can now start thinking of the plane that is "tangent" to the surface as a point.

Let's start out with a point (x_0, y_0) and let's let C_1 represent the trace to $f(x, y)$ for the plane $y = y_0$ (*i.e.* allowing x to vary with y held fixed) and we'll let C_2 represent the trace to $f(x, y)$ for the plane $x = x_0$ (*i.e.* allowing y to vary with x held fixed). Now, we know that $f_x(x_0, y_0)$ is the slope of the tangent line to the trace C_1 and $f_y(x_0, y_0)$is the slope of the tangent line to the trace C_2. So, let L_1 be the tangent line to the trace C_1 and let L_2 be the tangent line to the trace C_2.

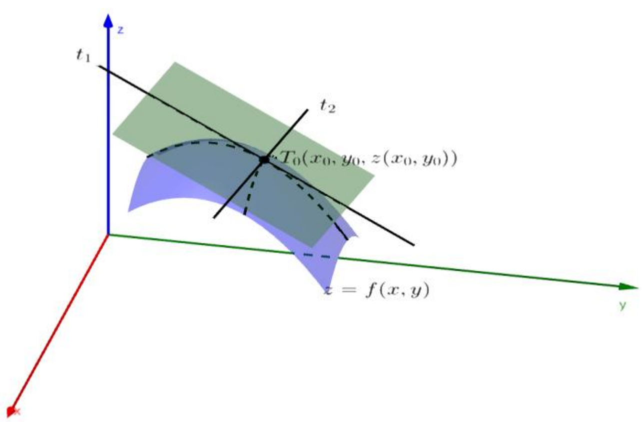
The tangent plane will then be the plane that contains the two lines L_1 and L_2.

Geometrically this plane will serve the same purpose that a tangent line did in Calculs I. A tangent line to a curve was a line that just touched the curve at that point and was "parallel" to the curve at the point in question. Well tangent planes to a surface are planes that just touch the surface at the point and are "parallel" to the surface at the point. Note that this gives us a point that is on the plane. Since the tangent plane and the surface touch at (x_0, y_0) the following point will be on both the surface and the plane.

$$(x_0, y_0, z_0) = (x_0, y_0, f(x_0, y_0))$$

What we need to do now is determine the equation of the tangent plane.

Tangent plane



We know that the general equation of a plane is given by,

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

where (x_0, y_0, z_0) is a point that is on the plane, which we have. Let's rewrite this a little. We'll move the x terms and y terms to the other side and divide both sides by c. Doing this gives,

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Action		
Materials / equipment / digital tools / software	*<u>The materials for learning</u>* are given as a part of references of the end from this topic plan; *<u>Equipment</u>*: classroom, whiteboard, marker in different colours; *<u>Digital tools</u>*: laptop, projector; *<u>Software</u>*: Geogebra, Mathematica.	
Consolidation	With the given examples students can consider that the applications of partial derivatives not only in geometry but in the filed of Economy. Students are encouraged to use technology, different digital tools and software as a help for solving problems, but can also realize that even with technology, solving different everyday problems is difficult without math knowledge.	

Reflections and next steps	
Activities that worked	**Parts to be revisited**
Problem solving, collaboration, using technology	Depends on the students, in a conversation with students the teacher will realize the difficulties that students had and then revisit appropriate parts.

References

[1] J. Stewart, Calculus, Thomson Learning, China, 2006.
[2] M. L. Bittinger, D. J. Ellenbogen and S.A. Surgent, “Calculus and its applications”, Addison-Wesley, 2012.
[3] T. Došenović, A. Takači, D. Rakić, Udžbenik iz Matematike II za studente Tehnološkog fakulteta, Univerzitet u Novom Sadu, 2017.
[4] https://tutorial.math.lamar.edu/classes/calciii/TangentPlanes.aspx